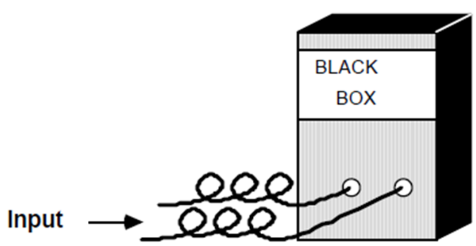
ECE-311 (ECE, NDSU)
Lab 4 – Experiment
Impedance 2: Input impedance and active circuits

1. Objective
The use of circuit simplification techniques and equivalent circuits are explored, leading to the concept of input impedance and output impedance. The description of an active electrical circuit in terms of impedances is also introduced.

2. Introduction
As considered in previous lab, the sinusoidal steady-state impedance is defined as the complex ratio of voltage to current. The basic circuit analysis procedures taught in EE-206 can be extended directly to the sinusoidal steady-state situation.
As you know, one common use of the impedance concept is in the simplification of complex electrical networks into a Thévenin equivalent impedance and voltage source. The main difference between the Thévenin circuit for a resistive network and the more general Thévenin "impedance" circuit is that the impedance varies with frequency.

a) <u>Input impedance</u>
Consider the situation in Figure 1. A circuit consisting of only unknown passive linear elements (resistors, capacitors, and inductors) is contained in a "black box" that is accessible only by two wires. Let's assume that the two wires represent the input to the circuitry contained in the black box (e.g., the input to an audio amplifier or the connections to an electric motor).



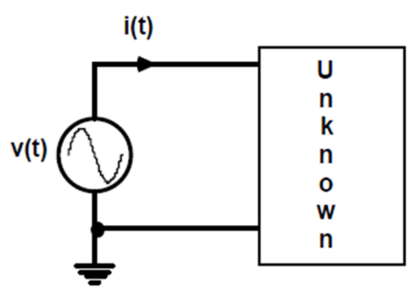


Figure 1: Input impedance

We want to determine the impedance in the box measured between the two input wires: the *input impedance*. One way to determine it is to connect a known voltage source between the inputs and measure the resulting input current; then complex ratio Zin = Vin/Iin.

Measurements should be made at a range of different input frequencies to determine the frequency dependent characteristics of the impedance. Note that in an actual measurement-setup a series resistor is typically included so that the input current can be displayed using an oscilloscope (as in previous lab). It is also important to note that this input impedance measurement procedure applies for passive networks only (no sources inside the black box).

b) Ouput impedance

Another similar situation, but now allowing an internal sinusoidal signal source, involves determination of the *output impedance* of an electrical network. The output impedance is defined as the Thévenin impedance of the network measured between the two output connections (see Figure 2).

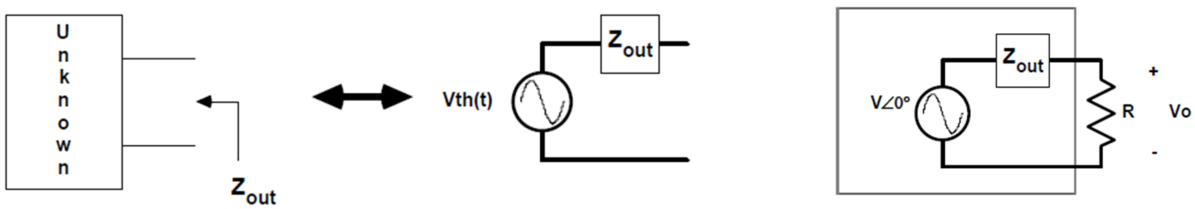


Figure 2: Output impedance

The Thévenin impedance (the output impedance in this case) is defined to be the complex ratio of the open circuit voltage and the short circuit current. In order to determine the Thévenin voltage we simply measure the open circuit voltage. However, the short circuit current presents several practical problems in attempting to determine the Thévenin impedance. First, we need the phase relationship between the voltage and current in the circuit, which means the two measurements must be made at the same time so that the relative phase can be determined. It is not possible to have both an open circuit and short circuit measurement at the same time. Second, even if we are only interested in the magnitude of the impedance and not the phase, it is often impractical to "short out" the circuit due to the large current that may flow if the output impedance is small.

So, how can we measure the Thévenin impedance Zout = Rout + jXout? Instead of using the short circuit current, we can make voltage and current measurements with two known load impedances attached. It is usually most convenient to use two resistors for the loads, since the voltage and current are in phase for resistances. Note that we can accomplish a similar measurement by attaching a known voltage source and measuring the resulting current for two different voltages, since the resistors we attach as loads obey Ohm's law.

Consider the circuit on the right hand-side of Figure 2. We assume that the Thévenin source magnitude |V| has already been determined by an open circuit voltage measurement, and the source phase is arbitrarily set to zero. Since in general we cannot have access to the components and nodes inside the box, we won't be able to determine the sign (impedance phase) of Zout by measuring the phase difference between V and Vo.
With known load resistors, R=R1 and R=R2, the magnitude of the output voltage can be expressed – in both situations – by the voltage division relationship. In this way, one arrives to

two equations involving |Vo|, Rout, and Xout. Note that we can measure |Vo| and therefore one will have left two equations with two unknowns: Rout and Xout, the resistive and reactive parts of the output impedance, respectively. With a bit of mathematical manipulations (*do it at home on your own; hint: subtract the above two equations*), one can derive the following expressions:

$$R_{out} = \frac{1}{2(R_1 - R_2)} \cdot \left[\left(R_1 \frac{|V|}{|V_{o1}|} \right)^2 - \left(R_2 \frac{|V|}{|V_{o2}|} \right)^2 \right] - \frac{(R_1 + R_2)}{2}$$

$$(X_{out})^2 = \left(R \frac{|V|}{|V_o|} \right)^2 - (R + R_{out})^2$$

If it were possible to determine the phase relationship between the Thévenin source and the output voltage we could determine the sign of Xout (i.e., whether the reactance was capacitive (-) or inductive (+)). Can you think of any other way we could determine the sign of Xout ?

c) ***Impedance and Active Circuits***

It is also possible to describe OpAmp circuits in terms of ac sinusoidal analysis. For example, consider the basic inverting OpAmp configuration in Figure 3.

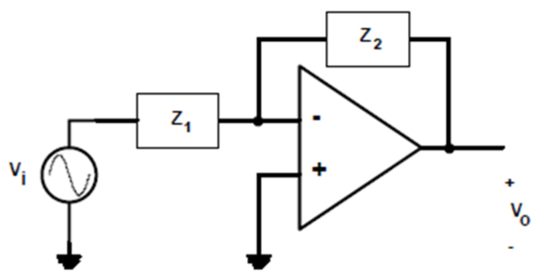


Figure 3: Inverter

Analysis of this circuit shows that the phasor relationship for the circuit is given by:

$$Vo = (-Z2/Z1) \cdot Vi$$

It is also possible to determine the input impedance and output impedance for the circuit. The input impedance (Vi/Ii) is simply Z1 because the input voltage appears entirely across Z1 (the inverting OpAmp input is held at ~0 volts). The output impedance is zero, since the output impedance of the OpAmp (ideal) is itself zero (parallel combination of zero with anything else is still zero). Of course, a real ApAmp is not completely ideal so the input and output impedances only approach these ideal values.

3. Experiment

(1) Assemble the circuit of Figure 4 using the function generator as the source and the nominal component values Ra= 1kΩ, Rb= 10kΩ, and C= 0.1μF. Make measurements of the *input*

impedance magnitude and phase at enough frequencies to fully describe the behavior of the circuit. You should do this using either one of the techniques learned in the previous lab.

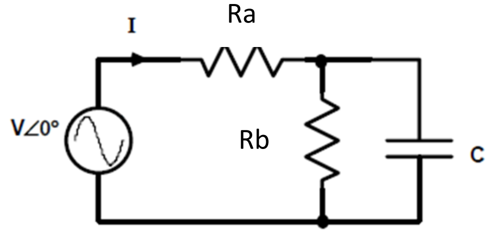


Figure 4: RC circuit

(2) Now, assume that the circuit of Figure 4 is an unknown circuit that you can access only via two long wires connected to the two terminals of the capacitor. Set the generator for 2V peak-to-peak at 2kHz. Using the "load resistor" approach described in the introduction (part 2.b above), make external measurements of your choice to come up with an estimate of the Thévenin equivalent circuit for the network as observed from the external connections.

(3) Assemble the circuit of Figure 5 using the indicated nominal component values. Measure the complex voltage gain (Vo/Vi), both magnitude and phase, over the frequency range 10Hz to 100kHz. Take sufficient measurements to fully describe the behavior of the circuit.

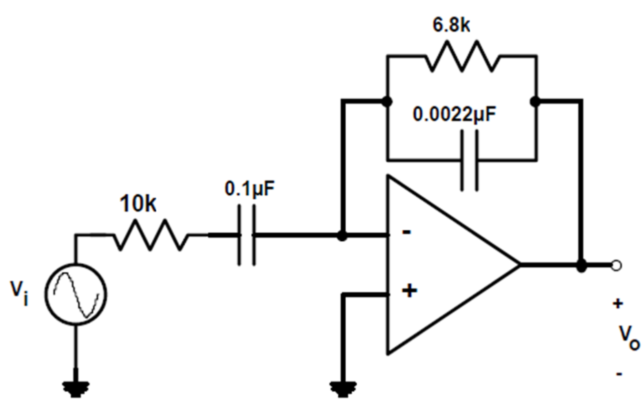


Figure 5: OpAmp circuit

4. Results

Your report should follow the template described in the lab syllabus.

(a) Prepare a plot of the impedance magnitude and phase using the measurements of part (1). Also, plot the theoretical prediction of the impedance using measured component values in the mathematical expression for the impedance (here you must derive manually, pen-and-paper, first an analytical expression of the impedance). Use a log scale for the frequency axis, plot the magnitudes on a log scale, and plot the phases on a linear scale. Discuss the results.

(b) What load resistor values did you use in your measurements for part (2)? Compare your measured Thévenin impedance results to a theoretical prediction based on the known circuit configuration.

(c) Plot your magnitude and phase measurements of the voltage gain from part (3) and compare with the mathematical expression (ideal OpAmp) and LTSpice simulations. Use a magnitude (log axis) vs. frequency (log axis) plot. How would you describe the agreement between the measurements and the predictions?

(d) How would you modify this experiment to make it more useful to you?