ECE-311 (ECE, NDSU)
Lab 14 – Experiment
Transformers

1. Objective

The characteristics of a small power transformer are compared to ideal and first-order transformer models. Measurements of transformer parameters and transformer applications are also examined, including a simple, unregulated DC power supply.

2. Introduction

A transformer is a four-terminal device in which AC complex power is applied at one pair of terminals (the *primary*) and a load is driven at the other pair of terminals (the *secondary*). The transformer is a magnetically coupled device with no direct electrical connection between the primary and the secondary windings, so no DC power can pass through the device. The behavior of an actual transformer is rather complicated, but several simplified models are available for analyzing basic transformer circuits.

The Ideal Transformer Model

An ideal transformer is lossless, meaning that the complex output power at the load is equal to the complex input power. A simple diagram representing the ideal transformer is shown in Figure 1.

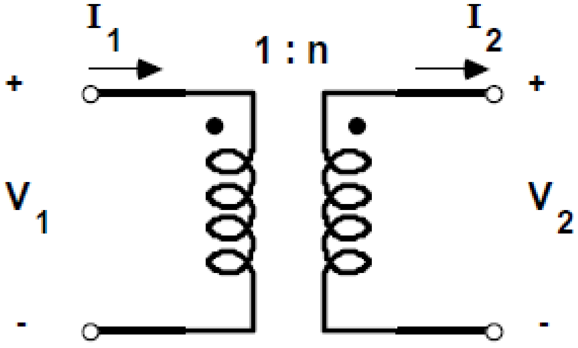

Figure 1

The ideal transformer can either *step up* or *step down* the AC output voltage (V_2) with respect to the input voltage (V_1) in order to obtain the desired voltage level at the load. Since no power is lost in the ideal transformer, stepping up the output voltage by a factor n results in $V_2 = n\,V_1$, and $I_2 = I_1/n$, where n is the *turns ratio* of the transformer, i.e., [# of turns in secondary winding]/[# of turns in primary winding]. The dots (•) at the top of the diagram in Figure 1 are used to indicate the polarity of the windings: a positive AC voltage V_1 results in a positive AC voltage V_2.

The typical application of transformers is depicted in Figure 2. In this example a sinusoidal source drives the primary and a load impedance is attached to the secondary.

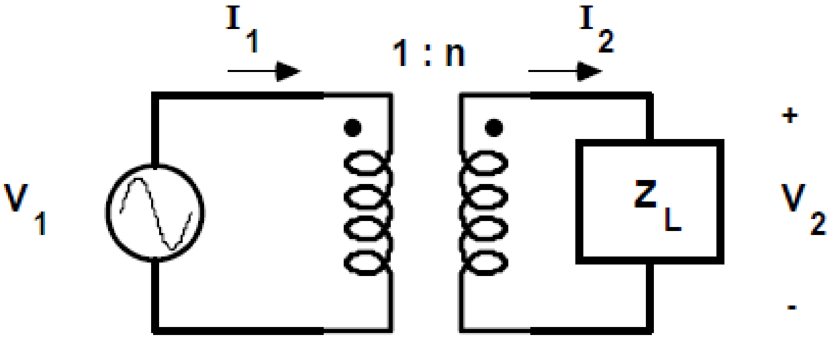

Figure 2

If we assume that the transformer is ideal we can identify the complex (phasor) expressions which relate the terminal voltages and currents.

$$nV_1 = V_2 \qquad I_1/n = I_2 \qquad V_2/I_2 = Z_L$$

Using these expressions we can also write

$$\begin{aligned} Z_1 &= \frac{V_1}{I_1} = \frac{V_2/n}{nI_2} \\ &= \frac{1}{n^2} \cdot Z_L \end{aligned}$$

This result indicates that for an ideal transformer the input impedance seen at the primary is equal to the impedance attached to the secondary divided by the square of the turns ratio. An equivalent circuit corresponding to this relationship is shown in Figure 3. It is also possible to determine the output impedance of this ideal transformer circuit (impedance seen by the load), which is n^2 times the source impedance. If an ideal voltage source is assumed, then the output impedance is zero.

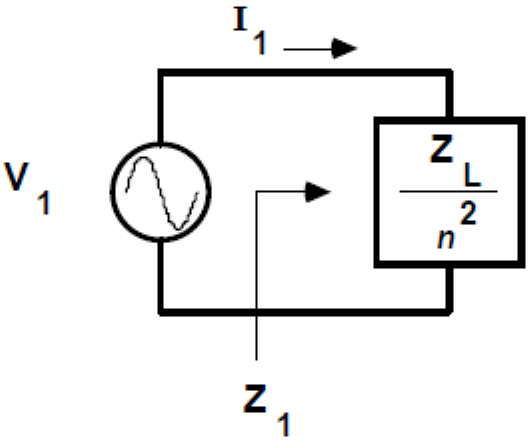

Figure 3

The impedance transformation relationship is interesting because it allows a transformer to be used in matching the impedance from one circuit to another.

A First-Order Low-Frequency Model

An actual transformer differs from the ideal model primarily due to the resistance and inductance of the windings and the incomplete coupling of magnetic flux between the primary and secondary windings. A model incorporating these effects is given in Figure 4

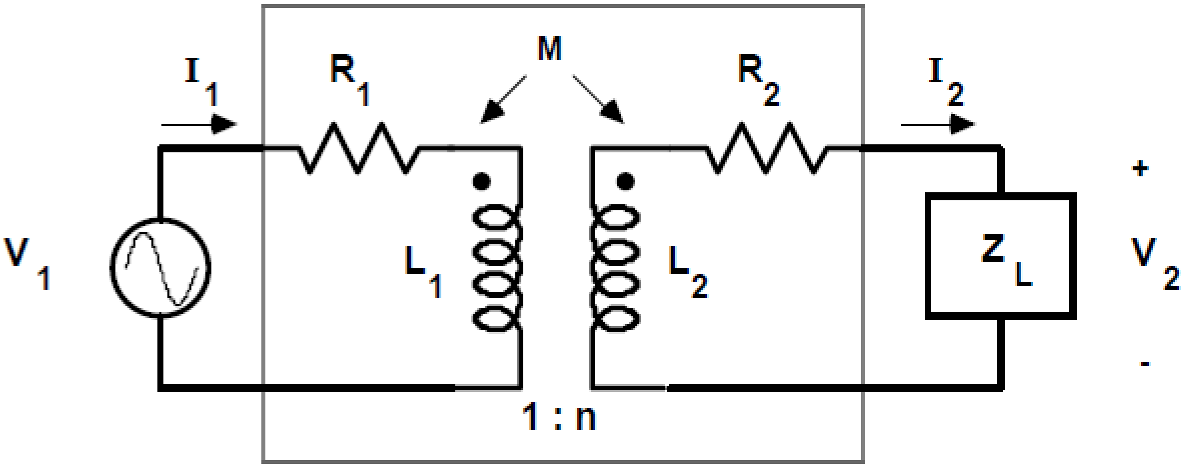

Figure 4

R_1, R_2, L_1 and L_2 model the resistive losses and inductance in the primary and secondary windings, respectively. The quantity M is the *mutual inductance* between the primary and secondary windings. The mutual inductance is given by

$$M = k \cdot \sqrt{L_1 L_2} ,$$

where k is the *coupling coefficient* of the particular transformer ($0 \leq k \leq 1$).

The basic equations for the transformer network of Figure 4 can be written using Kirchhoff's voltage law:

$$V_1 = (R_1 + j\omega L_1) \cdot I_1 - j\omega M \cdot I_2$$

$$V_2 = j\omega M \cdot I_1 - (R_2 + j\omega L_2) \cdot I_2 = Z_L \cdot I_2$$

The negative sign on the I_2 terms is due to our arbitrary definition of the secondary current as flowing out of the dotted terminal, while the primary current is defined to flow in. The parameters of this simple model can be measured as described below.

- The resistance terms R_1 and R_2 can be measured using an ohmmeter (DC) on the primary and secondary, respectively.

- If a voltage source is applied to the primary *while the secondary is left open-circuited ($I_2 = 0$)* the transformer equations reduce to:

  $$V_{1\,(oc)} = (R_1 + j\omega L_1) \cdot I_1$$

  $$V_{2\,(oc)} = j\omega M \cdot I_1$$

  If R_1 has been measured and we know the frequency ω, then measurements of $V_{1\,(oc)}$, $V_{2\,(oc)}$, and I_1 can be used to determine the value of L_1 and M.

- If a voltage source is now applied to the secondary *while the primary is left open-circuited ($I_1 = 0$)* the value of L_2 can be determined in a similar fashion.

A simple Line-Powered DC Supply

One common application of transformers is in power supply circuits. Many applications require either an AC or DC voltage source that must be derived from the available (in the USA) 110V rms 60 Hz sinusoidal voltage provided by the power company.

In addition to stepping up or down voltages, transformers also provide *electrical isolation* between the primary and secondary. Electrical isolation means that neither side of the secondary is connected to the primary power source. This is a very important safety consideration when powering electrical equipment from dangerously high voltages. In particular, isolation allows the secondary to receive its ground reference separately from the power source, thereby avoiding stray currents or unsafe power connections. The use of a third prong via the power plug is the recommended approach to provide safe grounding for the secondary.

A DC power supply can be created by peak rectifying an AC waveform. A simple low-voltage DC supply design utilizing a step-down transformer and a full-wave rectifier is shown in Figure 5.

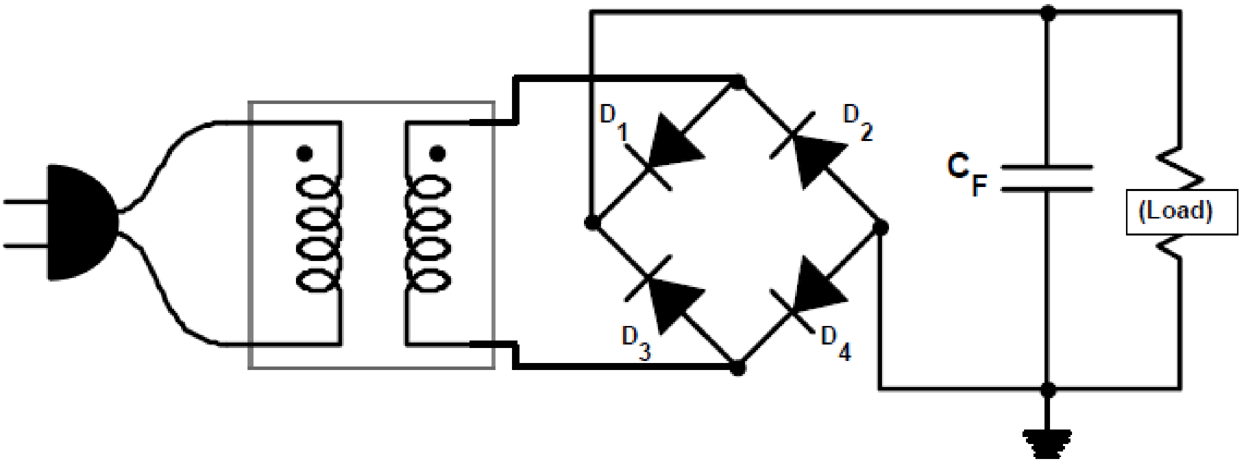

Figure 5

One problem with the supply of Figure 5 is that the output voltage across the load is not a pure DC value, but contains *ripple* due to the alternating charge-discharge cycles from the bridge rectifier. Furthermore, the circuit does not include any mechanism to maintain a constant output voltage as the load current varies or as the input line voltage fluctuates. To help solve these problems, a *voltage regulator* circuit is typically included between the filter capacitor, C_F, and the load in order to regulate the output voltage. Various types of voltage regulator circuits are available, including single integrated circuit devices.

3. Pre-lab Preparation

(I) A step down transformer has a 120V rms primary voltage applied and a 12V rms voltage is measured at the secondary. Assuming the transformer can be considered ideal, determine the turns ratio, n.

If a 1Ω load is attached to the secondary of this transformer what is the rms load current? What is the rms primary current?

(II) The windings of a particular transformer are made from 30-gauge copper wire. The primary resistance is found to be 10Ω and the secondary resistance is 4Ω. Estimate the length of wire in the primary and secondary,[*] and estimate the number of turns in each coil assuming each turn requires on average 5 cm of wire. Also calculate the turns ratio.

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[*] 30-gauge wire is 0.25 mm in diameter. Copper has a resistivity of 1.7×10^{-6} Ω cm. Can you figure out the resistance per unit length of the wire?

(III) The DC resistance of the primary windings of a certain transformer is found to be 22Ω, while the resistance of the secondary windings is 0.8Ω. When a 60 Hz sinusoidal source is connected to the primary *with the secondary open circuited* and the primary voltage is adjusted to be 15V peak-to-peak, the primary current *entering at the dot* is found to be 35mA peak-to-peak, lagging the voltage by 87°. The open circuit voltage on the secondary is measured with a high-impedance 'scope probe to be 1.5 volts peak-to-peak. Then, by driving the secondary with the primary open circuited and the secondary voltage adjusted to be 2V peak-to-peak, the secondary current *entering at the dot* is found to be 20mA peak-to-peak, lagging the voltage by nearly 90°. From these measurements determine an estimate for the self-inductances (L_1 and L_2), the mutual inductance (M), and the coupling coefficient (k) of the transformer.

4. Experiment

(1) The small power transformer used in this experiment is designed for 110V rms on the primary, producing a smaller rms on the secondary. Because in our laboratory we have two different types of transformers, first identify the rated voltage on the secondary.
Determine the primary and secondary winding resistances R_1 and R_2 by using the multimeter to measure the DC resistance of the primary and secondary.

(2) Assemble the circuit of Figure 6. Adjust the function generator for a 60 Hz sinusoid with amplitude sufficient to produce a 2V peak-to-peak signal across the 100Ω series resistor. Measure the voltage across the primary [A-B] and the voltage across the secondary [C-D] in order to determine the turns ratio, n.

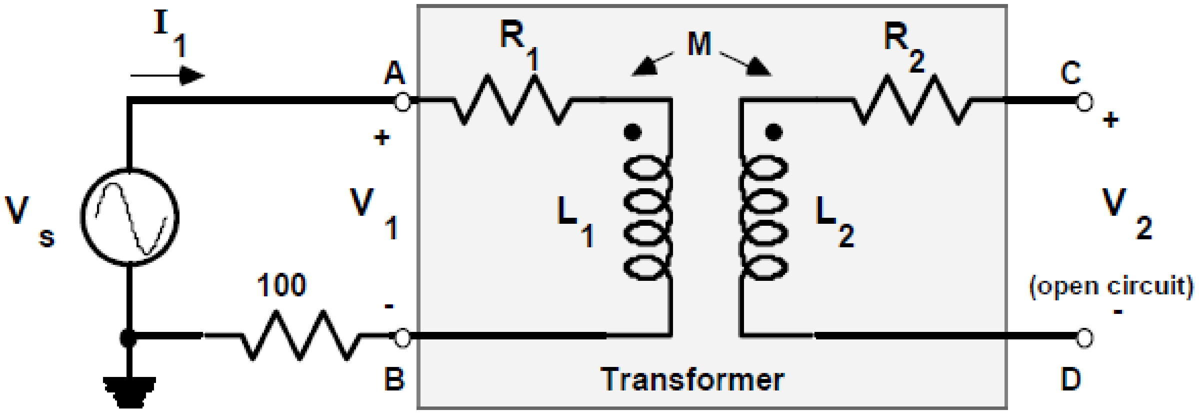

Figure 6

Use the phase measurement technique discussed in Lab #6 to determine the input impedance of the transformer with the secondary open-circuited (i.e., trigger from A, then [A-B] = primary voltage, B/100Ω = primary current, and the phase difference between [A-B] and B is the phase difference between the primary voltage and current). Also re-measure the open-circuit secondary voltage [C-D]. From these measurements determine an estimate for the primary self-inductance, L_1, and the mutual inductance, M.

Next, reconnect the circuit to drive the secondary, leaving the primary open-circuited, as shown in Figure 7. Keep in mind the defined direction of I_2. Make measurements to estimate the value of L_2 and M.

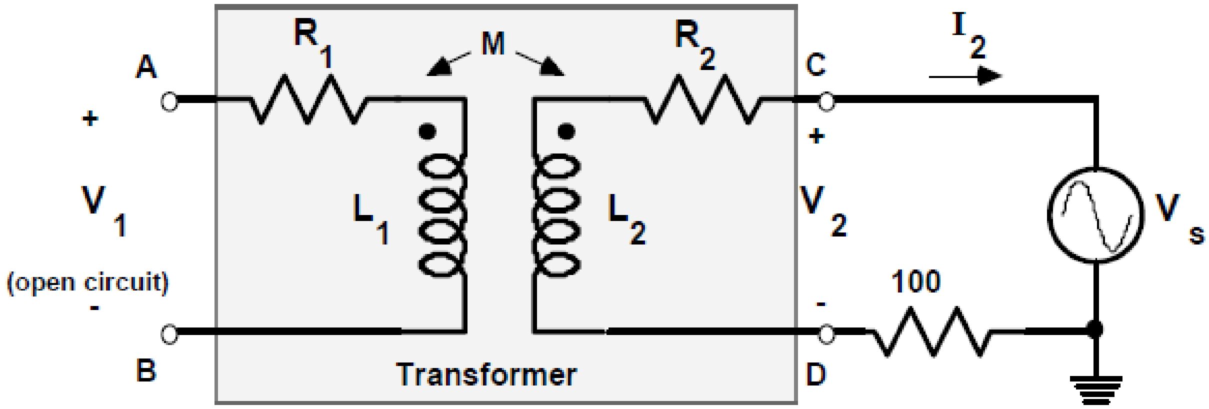

Figure 7

(3) ***Carefully measure the rms voltage of a standard 110V outlet***. Plug the primary of the transformer into the outlet and measure the secondary rms voltage. Verify that the secondary voltage is approximately the rms value identified in part 1. Now measure the secondary voltage for a range of resistors from your lab kit (be sure not to exceed the power dissipation limitation of the resistors!).

(4) Assemble the simple DC power supply circuit from the pre-lab, using several 22µF capacitors in parallel for the filter capacitor, C_F. ***Be sure to observe the correct polarity for the capacitors!*** Measure and sketch the ripple of the output waveform for a range of load resistances greater than 1kΩ, keeping in mind any effects due to the 1MΩ input resistance of the 'scope.

5. Results

(a) Present your measurements and calculations of the transformer parameters. Explain your findings. Do your measurements of L_1, L_2, and M seem reasonable? What is your estimate of the value of the coupling coefficient, *k*? Are the results consistent with your expectations? Comment on the accuracy of the measurement procedure.

(b) Describe your measurements of the secondary voltage as a function of the load resistance from part 3. Does the transformer behave like an ideal voltage source here?

(c) Summarize your findings for the simple DC power supply you assembled in part 4. Explain your ripple measurements.