ECE-311 (ECE, NDSU)
Lab 13 – Experiment
Two-port parameters

1. Objective

Two-port description of an electrical network is a useful analytical tool in electrical engineering. This lab considers the measurement issues related to two-port network descriptions, the relationship between the various two-port parameter definitions, and the use of equivalent two-port circuit models.

2. Background

A pair of terminals from a linear electrical network is referred to as a *port*. The voltage between the two terminals is the port voltage, and the current into the network is known as the port current. We require that Kirchhoff's current law be applicable at such a port, so whatever current enters the network on one of the port's terminals must exit the network on the other terminal. The conventional voltage and current definitions are indicated in Figure 1.

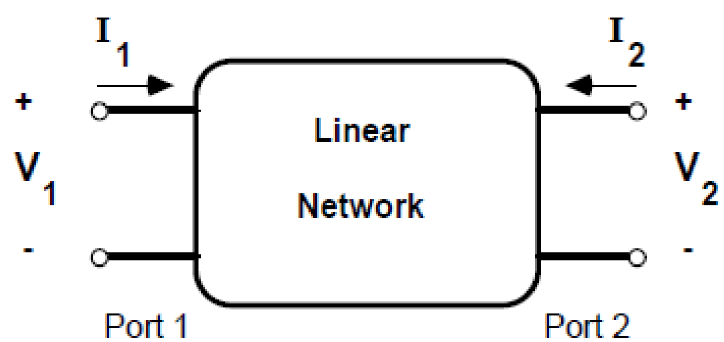

Figure 1

Note in particular that the direction of positive current is *into* the positive terminal of the port.

In general, a linear network can have an arbitrary number of ports. Since many problems in electrical engineering involve circuits which take an *input* signal and generate an *output* signal, we are typically most interested in *two-port* networks where one of the ports carries the input signal and the other port carries the output signal. In particular, the problem is to describe the mathematical relationships among the two port currents and the two port voltages.

Two-Port Parameters: the Basic Configurations

The voltage and current relationships for two-port networks can be expressed in five basic forms:

(1) I_1 and I_2 due to V_1 and V_2 (admittance form)
(2) V_1 and V_2 due to I_1 and I_2 (impedance form)
(3) V_1 and I_2 due to I_1 and V_2 (hybrid form)
(4) I_1 and V_2 due to V_1 and I_2 (inverse-hybrid form)
(5) V_1 and I_1 due to V_2 and I_2 (transmission form)

Our choice among the four forms to represent a given network is arbitrary because we can easily convert from one form to another.

The mathematical representations corresponding to the five two-port forms are given below. The subscripted parameters in the equations (y_{xy}, z_{xy}, etc.) are called the *two-port parameters* for the particular form of representation.

Admittance:

$$\begin{cases} I_1 = y_{11}V_1 + y_{12}V_2 \\ I_2 = y_{21}V_1 + y_{22}V_2 \end{cases} \quad \text{or} \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Impedance:

$$\begin{cases} V_1 = z_{11}I_1 + z_{12}I_2 \\ V_2 = z_{21}I_1 + z_{22}I_2 \end{cases} \quad \text{or} \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Hybrid:

$$\begin{cases} V_1 = h_{11}I_1 + h_{12}V_2 \\ I_2 = h_{21}I_1 + h_{22}V_2 \end{cases} \quad \text{or} \quad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Inverse-hybrid:

$$\begin{cases} I_1 = g_{11}V_1 + g_{12}I_2 \\ V_2 = g_{21}V_1 + g_{22}I_2 \end{cases} \quad \text{or} \quad \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

Transmission:

$$\begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases} \quad \text{or} \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

It is important to remember that because the elements in the linear two-port network are frequency-dependent in general (e.g., capacitors and inductors), the two-port parameters are also frequency-dependent. Thus, measurements of the two-port parameters will need to cover the desired range of response frequencies.

The parameters for a two-port network can be determined by applying pairs of excitation signals and measuring pairs of response signals. For example, if we wanted to determine the *impedance* parameter z_{11} for a two-port network we can solve the first port equation for z_{11} to yield

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$\Rightarrow \quad z_{11} = \frac{V_1 - z_{12}I_2}{I_1}.$$

If we let $I_2 = 0$, which means that we let port 2 be an open circuit (no current) during the measurement, then z_{11} is the complex ratio of V_1 and I_1. The z parameter measurements can be summarized:

- z_{11}: Set $I_2 = 0$ (open circuit), apply V_1 or I_1, and measure ratio $\left(z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \right)$
- z_{12}: Set $I_1 = 0$, apply V_2 or I_2, and measure ratio $\left(z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \right)$
- z_{21}: Set $I_2 = 0$, apply V_1 or I_1, and measure ratio $\left(z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \right)$
- z_{22}: Set $I_1 = 0$, apply V_2 or I_2, and measure ratio $\left(z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \right)$

Of course, a similar procedure can be developed for the other forms of two-port networks.

3. **Pre-lab preparation**

(I) Determine the *impedance* (z) and *hybrid* (h) two-port parameters for the network in Figure 2.

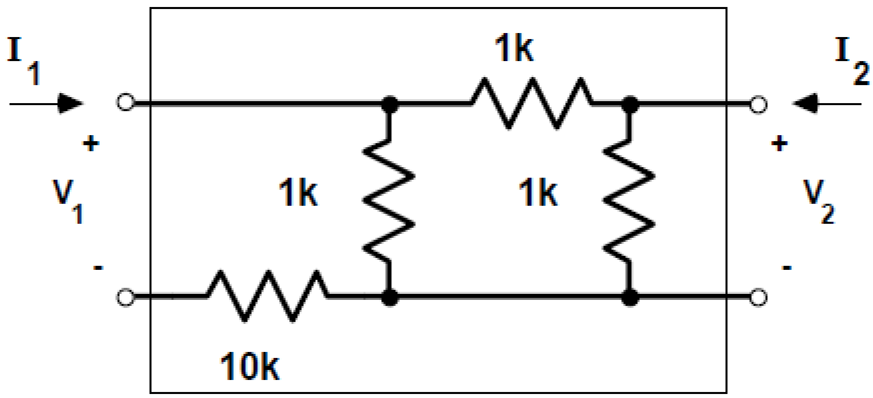

Figure 2

(II) Next, determine the *impedance* (z) and *hybrid* (h) two-port parameters for Figure 3.

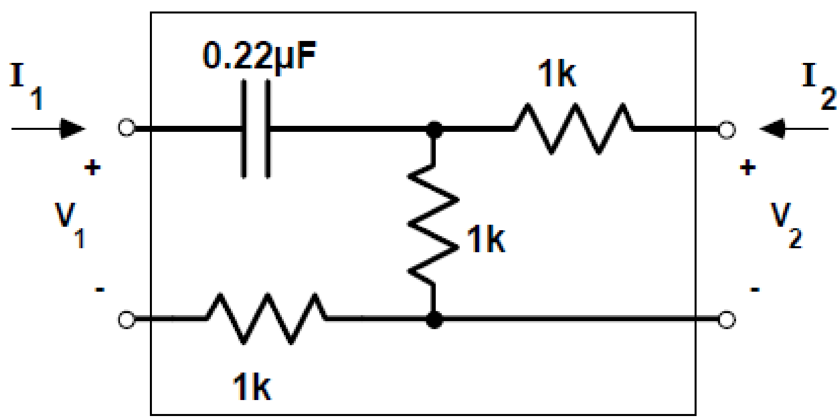

Figure 3

4. **Experiment**

(1) Assemble the circuit of Figure 2 and perform measurements to find the DC impedance parameters (z) of the network. Record the actual values of the components used.

(2) Assemble the circuit of Figure 3 and perform measurements of the AC (magnitude only) hybrid parameters (h) of the network at 1kHz, 2kHz and 5kHz. You will need to measure both AC voltages and currents, so think about how best to approach the measurements. Record the actual values of the components used.

5. Results

(a) Discuss your measured impedance parameters for part 1. How do your measurements compare to your expectations?

(b) Present your AC hybrid parameter measurements from part 2. Describe how you measured the magnitude of the signals. If the circuit was actually an unknown network sealed in a box and you only had access to the two ports, would you need to change your measurement technique? What if the *phase* of the hybrid parameters was to be measured?

SUPPLEMENTAL MATERIAL

NOTE: This section is optional and it is not required to be done as part of this lab. It is included here for additional information and experiments.

The Hybrid Parameters and Small-Signal Transistor Models

The hybrid two-port parameters involve exciting the network with the current I_1 an the voltage V_2, resulting in the voltage V_1 and the current I_2. This representation is similar to the active-mode behavior of bipolar junction transistor small-signal amplifiers, so AC hybrid parameters are often used to describe BJT operating characteristics. The small-signal simplified hybrid-π model of the BJT is shown in Figure 4, including the non-infinite output resistance, r_o.

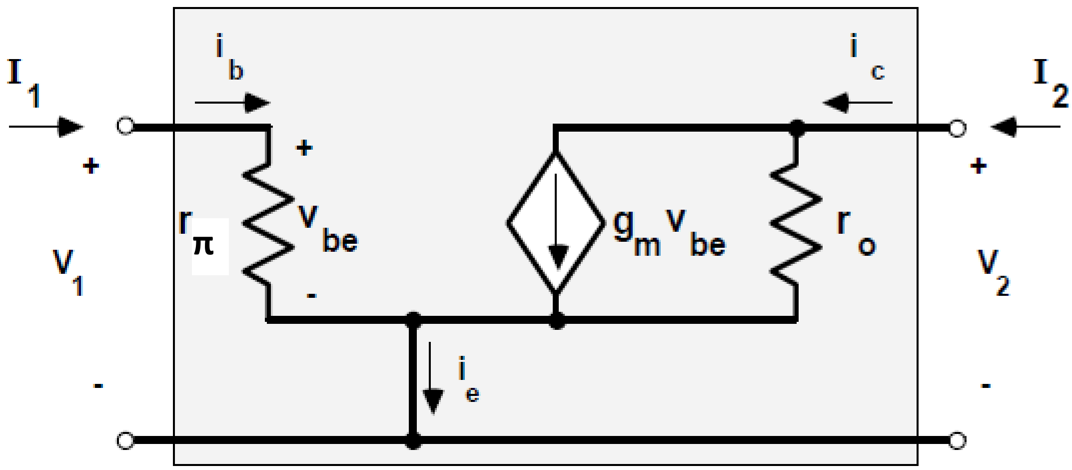

Figure 4

Note that the two ports share a common node at the emitter. Also, it is apparent that the port current I_1 is i_b and current I_2 is i_c in the BJT AC small-signal model. It is also clear that the port voltages V_1 and V_2 are v_{be} and v_{ce} in the model, respectively. Rewriting the hybrid equations,

$$\begin{cases} V_1 = h_{11}I_1 + h_{12}V_2 \\ I_2 = h_{21}I_1 + h_{22}V_2 \end{cases} \quad \text{becomes} \quad \begin{cases} v_{be} = h_{11}i_b + h_{12}v_{ce} \\ i_c = h_{21}i_b + h_{22}v_{ce} \end{cases}$$

Then, since we already know that for the BJT AC small-signal model $v_{be} = r_\pi i_b$ and $i_c = \beta i_b + v_{ce}/r_o$, the hybrid two-port parameters of the simple model in Figure 4 are given by

$$h_{11} = r_\pi, \qquad h_{12} = 0,$$
$$h_{21} = \beta, \quad \text{and} \quad h_{22} = 1/r_o.$$

The importance of this result is that if we can measure the hybrid two-port parameters of a biased BJT amplifier, then we can use the results to estimate the small-signal quantities of the transistor model.

It should also be noted that at high frequencies (f > 50kHz or so) the internal capacitance of the semiconductor junctions and packaging can become a significant factor in the small-signal model. At "low" frequencies (f < 10kHz) we will neglect the effects of capacitance and concentrate on the resistive elements of the model. A more complete AC small-signal model appropriate for the relatively low frequency range is shown in Figure 5. Note the inclusion of a resistance, r_{bb}, in series with the base and a base-collector resistor, r_μ. Resistor r_{bb} is used to model the nonzero resistance of the base material and electrical contacts, while r_μ models the effect of the collector voltage on the base. Since r_{bb} is typically much smaller than r_π, and r_μ is typically much greater than r_o, it is often OK to neglect these elements—as we have until now.

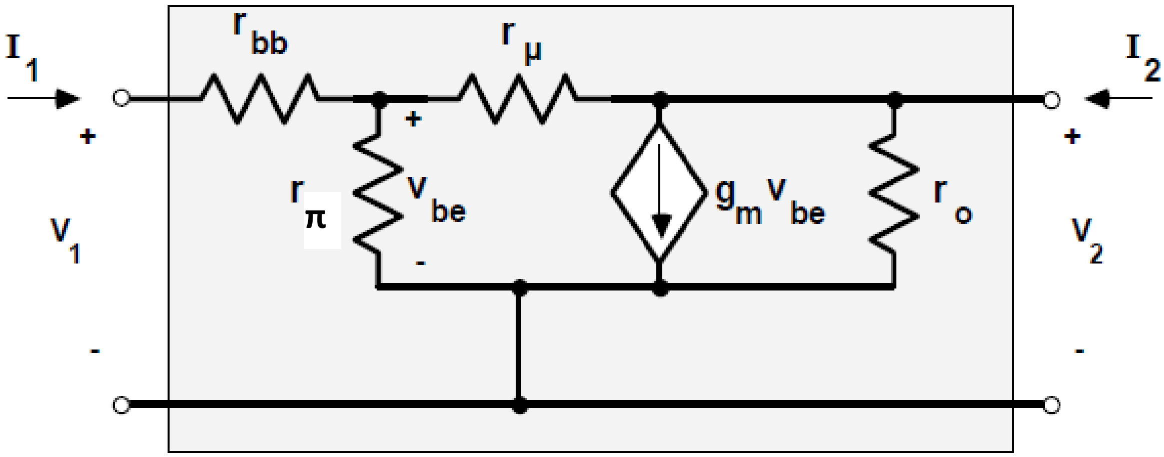

Figure 5

Manufacturers of transistors typically use the hybrid parameters on the data sheet to report the performance of BJTs. Instead of using the numerical subscripts, however, the conventional method is to express the hybrid equations as

$$\begin{cases} v_{be} = h_{ie} i_b + h_{re} v_{ce} \\ i_c = h_{fe} i_b + h_{oe} v_{ce} \end{cases},$$

where the subscript *i* indicates *input*, *o* indicates *output*, *f* indicates *forward*, *r* indicates *reverse*, and the common subscript *e* denotes the *common-emitter* port arrangement.* The data sheet generally indicates the conditions under which the parameters were determined, such as DC collector current and temperature.

* Note that the AC hybrid parameters use small-letter subscripts. Some manufacturers also report DC quantities using capital subscripts (such as forward current gain, h_{FE}).

If we include r_{bb} and r_μ in the hybrid parameter derivation we find the following relationships.

h_{ie}: From the definition $h_{ie} = h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$, we see that we need to find the input impedance at port 1 with a short-circuit ($V_2 = 0$) applied at port 2. The result is that $h_{ie} = r_{bb} + (r_\pi \| r_\mu)$. Note that if we assume $r_\pi << r_\mu$ and $r_\pi >> r_{bb}$, then $h_{ie} \approx r_\pi$ as was found for the simple equivalent circuit.

h_{re}: This parameter is defined $h_{re} = h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$, which means that we need to find the voltage V_1 due to V_2 with port 1 open circuited ($I_1 = 0$). This is simply the voltage divider of r_π and r_μ since no current flows in r_{bb}: $\mathbf{h_{re} = r_\pi/(r_\pi+r_\mu)}$. Again, if we assume that $r_\pi << r_\mu$ then $h_{re} \varnothing 0$ as we found before.

h_{fe}: The short-circuit current gain is defined $h_{fe} = h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$. Writing node equations for I_2 and I_1:

$$I_2 = g_m v_{be} - v_{be}/r_\pi$$
$$I_1 = v_{be}/(r_\pi \| r_\mu)$$

resulting in

$$h_{fe} = \frac{g_m - 1/r_\pi}{1/\left(r_\pi \| r_\mu\right)} = \frac{g_m r_\mu r_\pi - r_\pi}{r_\mu + r_\pi} = \frac{\beta r_\mu - r_\pi}{r_\mu + r_\pi}.$$

Note that $h_{fe} \approx \beta$ if we again assume that $r_\pi << r_\mu$.

h_{oe}: Finally, using the definition $h_{oe} = h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ and equation for I_2 in terms of V_2 (with port 1 open circuited):

$$I_2 = V_2 \cdot [\, 1/r_o + 1/(r_\pi+r_\mu) + g_m r_\pi/(r_\pi+r_\mu) \,],$$

we have h_{oe} = [$1/r_o + 1/(r_\pi+r_\mu) + \beta/(r_\pi+r_\mu)$], or $h_{oe} \approx 1/r_o + \beta/(r_\pi+r_\mu)$ if $\beta >> 1$. As before, if r_μ is assumed to be much greater than r_π then $h_{oe} \approx 1/r_o + \beta/r_\mu \approx 1/r_o$.

Collecting all the relationships we can solve for the BJT small-signal model parameters using the measured DC bias collector current, I_C, and the measured h-parameters:

$$g_m = \frac{I_C}{V_T} \qquad (V_T \approx 25\text{mV})$$

$$r_\pi = \frac{h_{fe}}{g_m} \qquad\qquad r_\mu = \frac{r_\pi}{h_{re}}$$

$$r_o = \frac{1}{\left(h_{oe} - \frac{h_{fe}}{r_\mu}\right)} = \frac{V_A}{I_C} \qquad\qquad r_{bb} = h_{ie} - r_\pi$$

Note that r_o is most easily determined from V_A/I_C.

Measurement Issues for BJT Two-Port Parameters

In order to actually measure the hybrid parameters for a biased BJT it is important to avoid changing the DC bias conditions during the measurement. For example, to determine h_{ie} from the equation $v_{be} = h_{ie}\, i_b + h_{re}\, v_{ce}$ we may be tempted to simply force v_{ce} to be zero by shorting the collector to the emitter, then measure $h_{ie} = v_{be}/i_b$. However, shorting the collector to the emitter saturates the transistor and destroys the DC bias conditions. Instead, we must use an "AC short circuit", e.g., a capacitor, from collector to emitter so that the DC bias conditions are maintained, while the AC model "sees" zero volts across port 2.

Supplemental pre-lab preparation

(III) In order to measure the small-signal h-parameters for a biased BJT we will use the two circuits of Figure 6a and b.

(IV) Sketch the AC small-signal models of Figures 6a and b. Label the components, nodes, currents, and voltages.

- Choose resistors R_B and R_C from the 5% tolerance resistors (or combinations) in your kit so that the DC bias conditions are $I_C \approx 10mA$ and $V_C \approx 9$ volts, assuming $V_{BE} = 0.7$ volts, $V_{CC} = 18$ volts, and $\beta = 100$.

- For the measurements on Figure 6a, we want $R_S << R_B$ and $R_L << R_C$ so that the AC small-signal current in R_S is essentially i_b and the small-signal current in R_L is essentially i_c. Thus, choose R_L to be about $R_C/10$, and R_S to be about $R_B/10$. Use 22µF capacitors for C_B and C_C.

  Now show how you can determine expressions for the AC small-signal BJT parameters h_{fe} and h_{ie} from measurements of v_x, v_b, and v_o in the circuit of Figure 6a.

- The circuit of Figure 6b can be used to determine the AC small-signal BJT parameter h_{re}, assuming that R_B and the oscilloscope impedance are both >> r_π so that the base "sees" an open circuit (port 1 current must be zero for h_{re}). Find the simple expression for h_{re} in terms of AC voltages from the Figure.

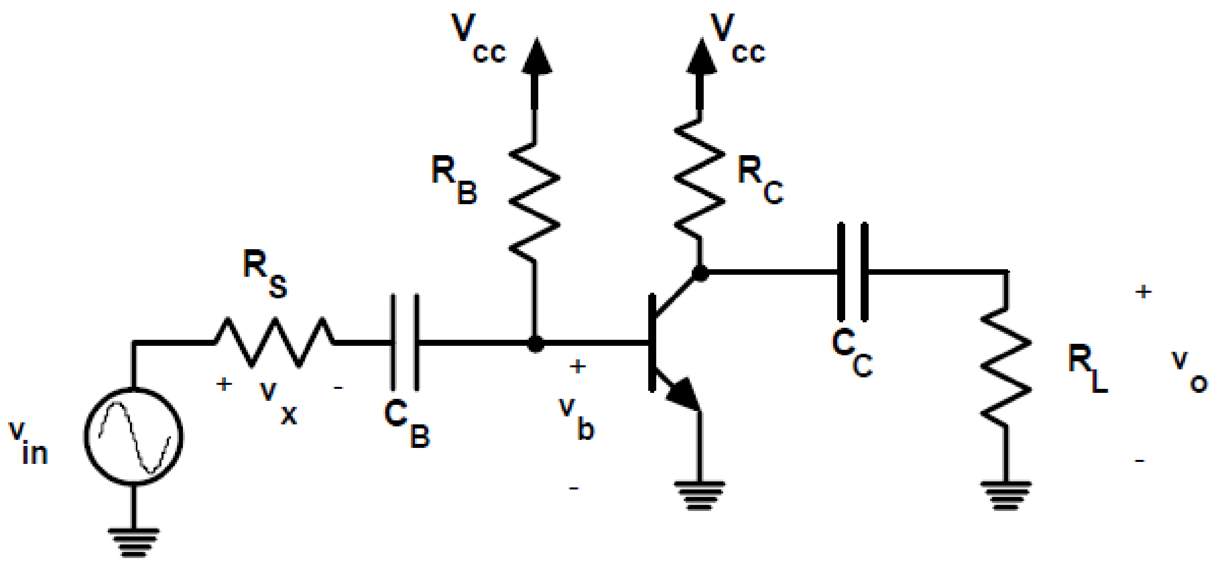

Figure 6a

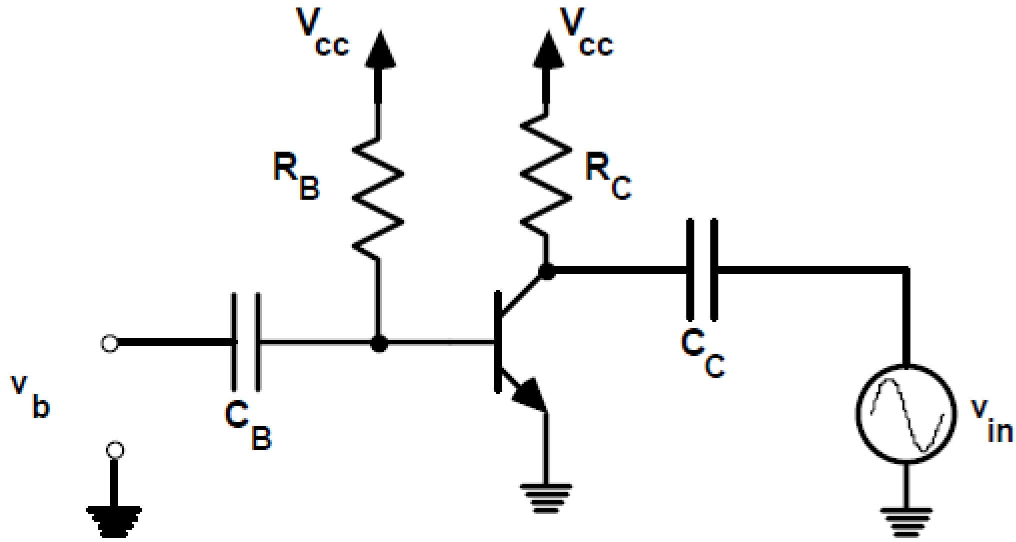

Figure 6b

Supplemental experiment

(3) Use the curve tracer instrument in the lab to obtain a plot of I_C vs. V_{CE} for your BJT. From the measurement estimate the value of β and V_A.

(4) Using the measured value of β, choose the four resistors for the circuit of Figure 6a using the guidelines as you did in the pre-lab. Assemble the circuit and adjust R_C and R_B as necessary to obtain $I_C \approx 10$ mA and $V_C \approx 9$ volts. Adjust the function generator for an AC small signal input to the circuit at 10kHz. Use the scope (AC coupling) or the DMM (AC) to measure v_x, v_b, and v_o in order to estimate h_{fe} and h_{ie} using the expressions you determined in the pre-lab.

(5) Now keep the same values of R_B and R_C and reconnect your circuit according to Figure 6b. Carefully measure the required quantities to estimate h_{re}.

Supplemental results

(c) Explain your measurements of parts 3 - 5 and present your estimates of the small-signal model parameters r_{bb}, r_π, g_m, r_μ, and r_o. Do your model parameters make sense? What types of errors might be involved in your measurements, e.g., can r_{bb} and r_μ be determined accurately?