ECE-311 (ECE, NDSU)
Lab 11 – Experiment
Multi-stage RC low pass filters

1. Objective

In this lab you will use the single-stage RC circuit filter to build a 3-stage RC low pass filter. The objective of this lab is to show that:

- As more stages are added, the filter becomes able to better reject high frequency noise
- When plotted on a Bode plot, the gain approaches two asymptotes: the low frequency gain approaches a constant gain of 0dB while the high-frequency gain drops as 20N dB/decade where N is the number of stages.

2. Background

The single stage RC filter is a low pass filter: low frequencies are passed (have a gain of one), while high frequencies are rejected (the gain goes to zero). This is a useful filter to remove noise from a signal. Many types of signals are predominantly low-frequency in nature - meaning they change slowly. This includes measurements of temperature, pressure, volume, position, speed, etc. Noise, however, tends to be at all frequencies, and is seen as the "fuzzy" line on you oscilloscope when you amplify the signal.

The trick when designing a low-pass filter is to select the RC time constant so that the gain is one over the frequency range of your signal (so it is passed unchanged) but zero outside this range (to reject the noise).

3. Theoretical response

One problem with adding stages to an RC filter is that each new stage ***loads*** the previous stage. This loading consumes or "bleeds" some current from the previous stage capacitor, changing the behavior of the previous stage circuit. If we design the newly added stage such that its impedance is 10 times that of the previous stage, this loading is less than 10%, and we could ***approximate*** the transfer function of each stage with that of a single stage RC circuit in isolation.

The three stage RC low pass filter from Fig. 1 is an example of such a circuit.

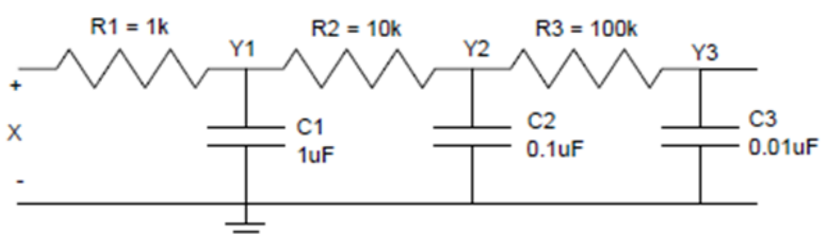

Figure 1: Multi-stage low pass filter

The gain of this filter is approximately the gain of each stage analyzed separately (i.e., the loading effects are ignored).

$$Y_1 \approx \left(\frac{1}{j\omega R_1 C_1+1}\right) X$$

$$Y_2 \approx \left(\frac{1}{j\omega R_2 C_2+1}\right) Y_1 = \left(\frac{1}{j\omega RC+1}\right)^2 X$$

$$Y_3 \approx \left(\frac{1}{j\omega R_3 C_3+1}\right) Y_2 = \left(\frac{1}{j\omega RC+1}\right)^3 X$$

If you use Matlab to plot the gain vs. frequency you should get the plots shown in Fig. 2:

```
w = logspace(2,4,100)'; % select 100 points from 100 to 10k rad/sec
G1 = 1 ./ (j*w*0.001+1); % gain of a single stage, RC=0.001
G2 = G1 .^2; % gain of two stages
G3 = G1 .^3;
semilogx(w, 20*log10(abs(G1)));
```

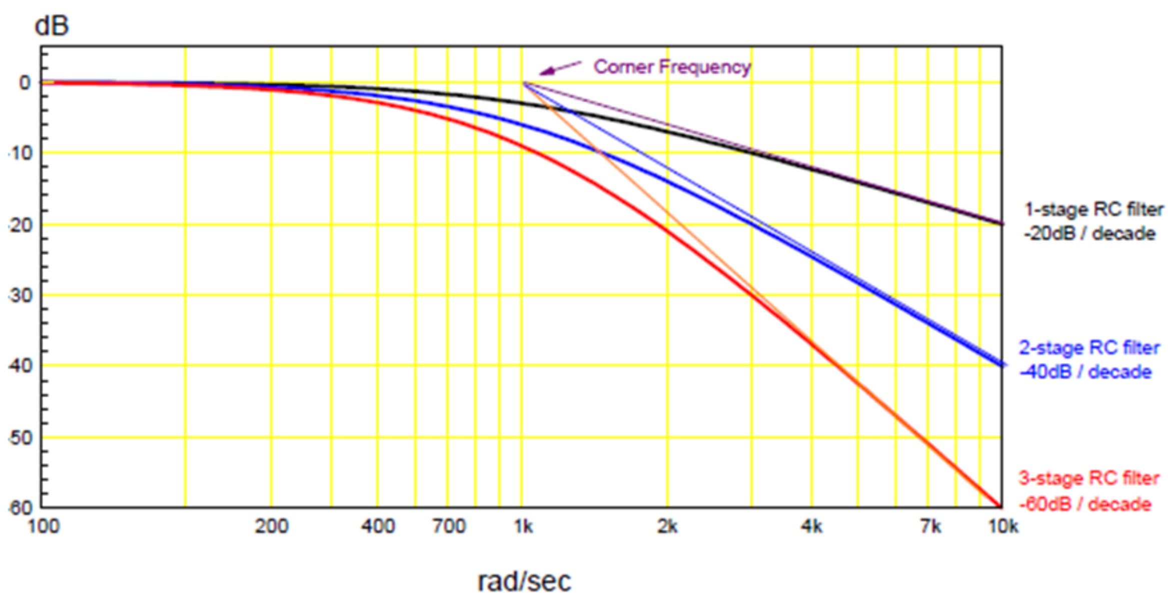

Figure 2: Magnitude Bode plots

On this plot, several interesting features should be observed:

- The low-frequency gain is a constant, 0dB (1)
- The high-frequency gain approaches an asymptote with a slope of -20N dB/decade. When the frequency increases 10 times, the gain drops 10^N times. 10^N = 20N dB, where N is the number of stages.
- The two asymptotes intersect at 1/RC rad/sec or 1/2πRC Hz. This is called the ***corner frequency*** (a.k.a. ***cutoff frequency***) since the asymptotes intersect at a corner.
- The three stage RC filter has a gain which drops much faster than a one stage RC filter.

4. Procedure

(1) Build a 3-stage RC filter shown above. Let the corner frequency be 166Hz (1000 rad/sec).

(2) Measure the gain of a 1-stage, 2-stage, and 3-stage RC filter from this filter from 100 to 10k rad/sec (16Hz to 1.6kHz)

(3) Plot the gain vs. frequency on a Bode plot for a 1-stage, 2-stage, and 3-stage RC low pass filter. On these plots, show the asymptotes and verify that the gain does behave as the transfer function predicts.

5. Lab report

(1) Compare the actual gain and phase shift of your filter to what the theoretical analysis predicts.

(2) Determine what the output of a 1-stage, 2-stage, and 3-stage RC low pass filter is when the input signal is a 1Hz sine wave with 1kHz noise on top:

$$x(t) = 5\cos(6.28t) + 4\cos(6280t)$$