

Lecture 3
Combinational and Sequential Circuits I

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Overview

- **Combinational circuits**
 - Multiplexer, decoders, encoders, adders, comparators
- **Sequential circuits**
 - Regular sequential circuits
 - Finite State Machines

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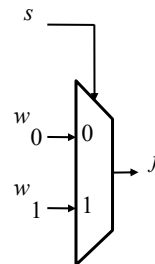
A VHDL Template for Combinational Logic

```
entity model_name is  
    port( list of inputs and outputs );  
end model_name;  
  
architecture arch_name of model_name is  
begin  
    concurrent statement 1  
    concurrent statement 2  
    ...  
    concurrent statement N;  
end arch_name;
```

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2-to-1 Multiplexer

```
LIBRARY ieee;  
USE ieee.std_logic_1164.all;  
  
ENTITY mux2to1 IS  
    PORT ( w0, w1, s : IN STD_LOGIC;  
          f : OUT STD_LOGIC);  
END mux2to1;  
  
ARCHITECTURE dataflow OF mux2to1 IS  
BEGIN  
    f <= w0 WHEN s = '0' ELSE w1;  
END dataflow ;
```



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4-to-1 Multiplexer

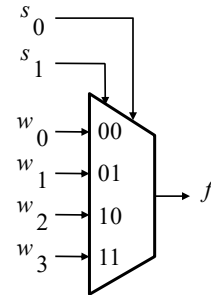
```

LIBRARY ieee;
USE ieee.std_logic_1164.all;

ENTITY mux4to1 IS
    PORT ( w0, w1, w2, w3: IN STD_LOGIC;
          s: IN STD_LOGIC_VECTOR(1 DOWNTO 0);
          f: OUT STD_LOGIC );
END mux4to1 ;

ARCHITECTURE dataflow OF mux4to1 IS
BEGIN
    WITH s SELECT
        f <= w0 WHEN "00",
           w1 WHEN "01",
           w2 WHEN "10",
           w3 WHEN OTHERS;
END dataflow

```



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2-to-4 Decoder

```

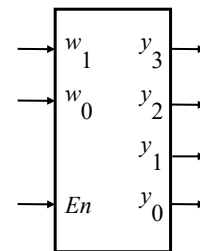
LIBRARY ieee;
USE ieee.std_logic_1164.all;

ENTITY dec2to4 IS
    PORT (w : IN STD_LOGIC_VECTOR(1 DOWNTO 0);
          En : IN STD_LOGIC;
          y : OUT STD_LOGIC_VECTOR(3 DOWNTO 0));
END dec2to4 ;

ARCHITECTURE dataflow OF dec2to4 IS
    SIGNAL Enw : STD_LOGIC_VECTOR(2 DOWNTO 0);
BEGIN
    Enw <= En & w ;
    WITH Enw SELECT
        y <= "0001" WHEN "100",
           "0010" WHEN "101",
           "0100" WHEN "110",
           "1000" WHEN "111",
           "0000" WHEN OTHERS;
END dataflow;

```

En	w_1	w_0	y_3	y_2	y_1	y_0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0
0	x	x	0	0	0	0



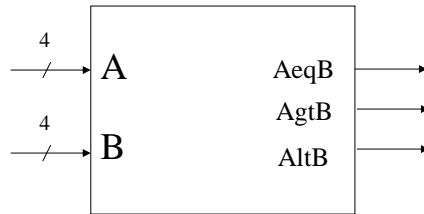
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4-bit Number Comparator: Unsigned

```
LIBRARY ieee ;
USE ieee.std_logic_1164.all ;
USE ieee.std_logic_unsigned.all ;

ENTITY compare IS
    PORT (A, B: IN STD_LOGIC_VECTOR(3 DOWNTO 0);
          AeqB, AgtB, AltB: OUT STD_LOGIC );
END compare;

ARCHITECTURE dataflow OF compare IS
BEGIN
    AeqB <= '1' WHEN A = B ELSE '0';
    AgtB <= '1' WHEN A > B ELSE '0';
    AltB <= '1' WHEN A < B ELSE '0';
END dataflow;
```



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4-bit Number Comparator: Signed

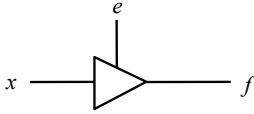
```
LIBRARY ieee;
USE ieee.std_logic_1164.all;
USE ieee.std_logic_signed.all;

ENTITY compare IS
    PORT (A, B: IN STD_LOGIC_VECTOR(3 DOWNTO 0);
          AeqB, AgtB, AltB: OUT STD_LOGIC );
END compare;

ARCHITECTURE dataflow OF compare IS
BEGIN
    AeqB <= '1' WHEN A = B ELSE '0';
    AgtB <= '1' WHEN A > B ELSE '0';
    AltB <= '1' WHEN A < B ELSE '0';
END dataflow;
```

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Tri-state Buffer



e	x	f
0	0	Z
0	1	Z
1	0	0
1	1	1

```

ENTITY tri_state IS
  PORT ( e: IN STD_LOGIC;
        x: IN STD_LOGIC;
        f: OUT STD_LOGIC);
END tri_state;

```

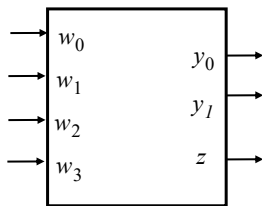
```

ARCHITECTURE dataflow OF tri_state IS
BEGIN
  f <= x WHEN (e = '1') ELSE 'Z';
END dataflow;

```

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Priority Encoder



w ₃	w ₂	w ₁	w ₀	y ₁	y ₀	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

```

LIBRARY ieee ;
USE ieee.std_logic_1164.all ;

ENTITY priority IS
  PORT ( w: IN STD_LOGIC_VECTOR(3 DOWNTO 0);
        y: OUT STD_LOGIC_VECTOR(1 DOWNTO 0);
        z: OUT STD_LOGIC );
END priority ;

```

```

ARCHITECTURE dataflow OF priority IS
BEGIN
  y <= "11" WHEN w(3) = '1' ELSE
      "10" WHEN w(2) = '1' ELSE
      "01" WHEN w(1) = '1' ELSE
      "00";
  z <= '0' WHEN w = "0000" ELSE '1';
END dataflow ;

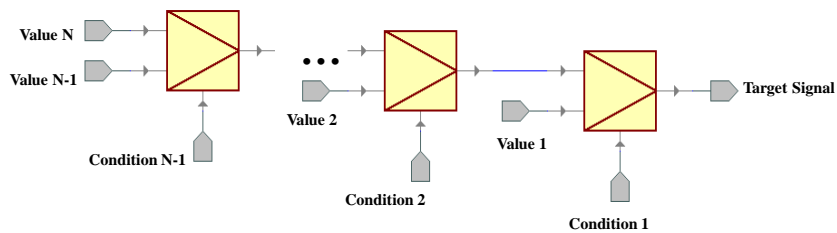
```

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Most often implied structure

When - Else

```
target_signal <= value1 when condition1 else  
                value2 when condition2 else  
                . . .  
                valueN-1 when conditionN-1 else  
                valueN;
```

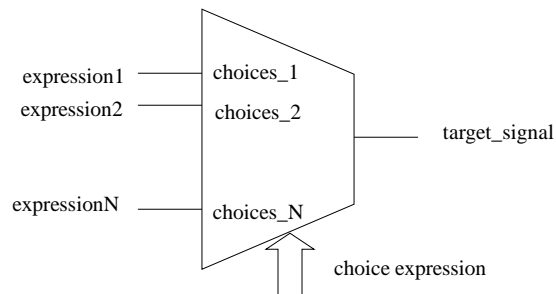


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Most often implied structure

With - Select - When

```
with choice_expression select  
  target_signal <= expression1 when choices_1,  
                    expression2 when choices_2,  
                    . . .  
                    expressionN when choices_N;
```



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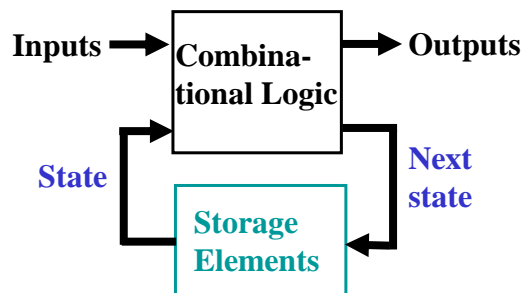
Sequential Circuits

- Regular sequential circuits
 - Sequential circuits
 - Storage elements: Latches & Flip-flops
 - Registers and counters
- Circuit and System Timing
- Finite State Machines (FSMs)
 - State tables & state diagrams

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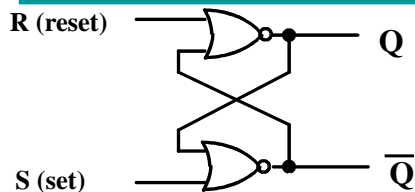
Sequential circuits – general description

- A Sequential circuit contains:
 - Storage elements: Latches or Flip-Flops
 - Combinational Logic: implements a multiple-output switching function
 - *Next state function:* Next State = $f(\text{Inputs}, \text{State})$
 - *Output function: two types*
 - Mealy: Outputs = $g(\text{Inputs}, \text{State})$
 - Moore: Outputs = $h(\text{State})$

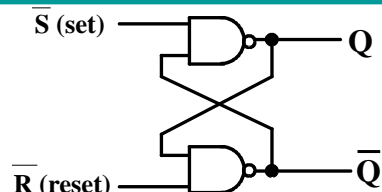


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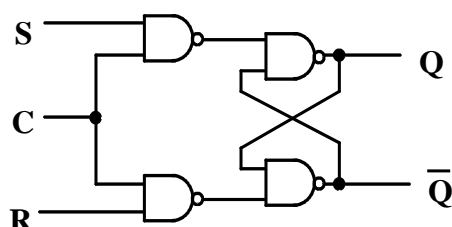
Latches



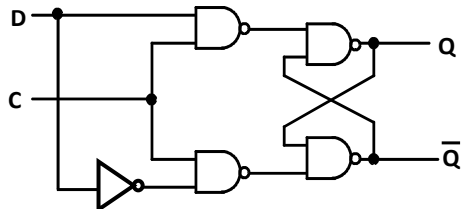
Basic S-R latch



Basic \bar{S} - \bar{R} latch



Clocked S-R latch

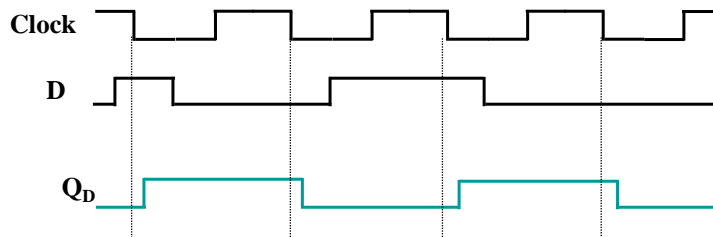
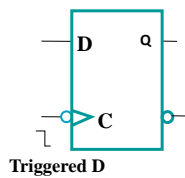
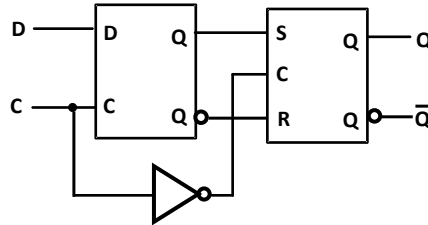


D latch

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Edge-Triggered D Flip-Flop

- The change of Q is associated with the negative edge at the end of the pulse - *negative-edge triggered* flip-flop.



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Modelling of Flip-Flops

```
Library IEEE;
use IEEE.Std_Logic_1164.all;

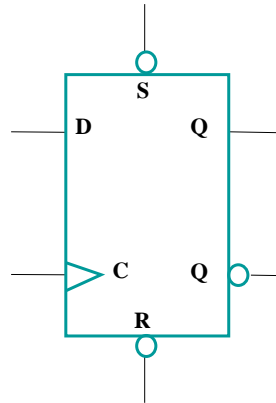
entity FLOP is
    port (D, CLK :in std_logic;
          Q : out std_logic);
end FLOP;

architecture A of FLOP is
begin
    process
    begin
        wait until CLK'event and CLK='0';
        Q <= D;
    end process;
end A;
```

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Direct Inputs

- At power-up or at reset, sequential circuit usually is initialized to a known state before it begins operation
 - Done outside of the clocked behavior of the circuit (i.e., asynchronously).
 - Direct R/S inputs
- For the example flip-flop shown
 - 0 applied to R resets the flip-flop to the 0 state
 - 0 applied to S sets the flip-flop to the 1 state



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Positive Edge-triggered D Flip-flop with Asynchronous Set/Reset

```
library IEEE;
use IEEE.std_logic_1164.all;

entity ASYNC_FF is
    port (D, CLK, SETN, RSTN : in std_logic;
          Q : out std_logic);
end ASYNC_FF;

architecture RTL of ASYNC_FF is
begin
    process (CLK, RSTN, SETN)
    begin
        if (RSTN = '1') then
            Q <= '0';
        elsif SETN = '1' then
            Q <= '1';
        elsif (CLK'event and CLK = '1') then
            Q <= D;
        end if;
    end process;
end RTL;
```

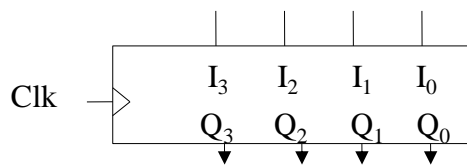
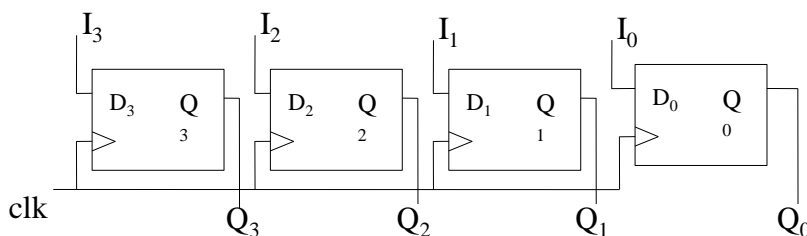
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Registers

- Register: the simplest storage component in a computer, a bit-wise extension of a flip-flop.
- Registers can be classified into
 - Simple Registers
 - Parallel-Load Registers
 - Shift Registers

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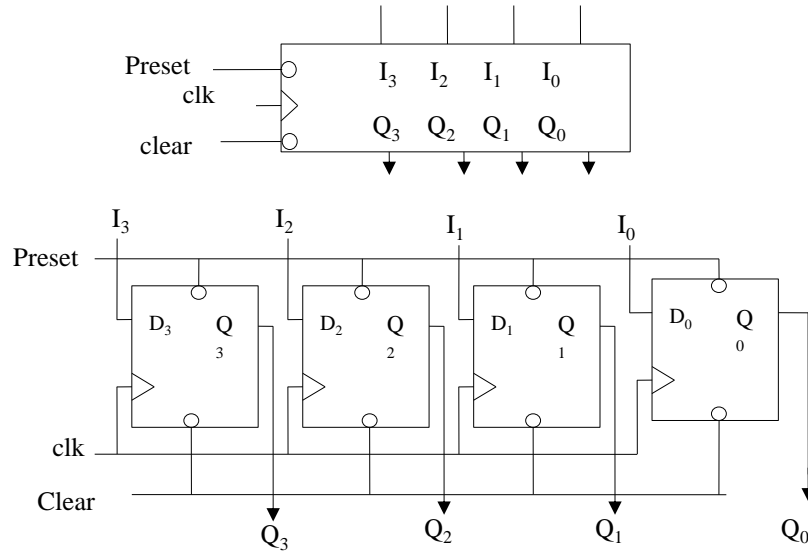
Simple Registers



- A simple register consists of N flip-flops driven by a common clock signal.
- Has N inputs and N outputs in addition to the clock signal.

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Register with asynchronous preset and clear



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```

Library ieee;
USE ieee.std_logic_1164.all;
ENTITY simple_register IS
    GENERIC ( N : INTEGER := 4);
    PORT ( I : IN STD_LOGIC_VECTOR (N-1 DOWNTO 0);
          Clock, Clear, Preset : IN STD_LOGIC;
          Q : OUT STD_LOGIC_VECTOR (N-1 DOWNTO 0));
END simple_register;

ARCHITECTURE simple_memory OF simple_register IS
BEGIN
    PROCESS (Preset, Clear, Clock)
    BEGIN
        IF Preset = '0' THEN
            Q <= (OTHERS => '1');
        ELSIF Clear = '0' THEN
            Q <= (OTHERS => '0');
        ELSIF (Clock'EVENT AND Clock = '1') THEN
            Q <= I;
        END IF;
    END PROCESS;
END simple_memory;
    
```

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Parallel Load Registers

- In the previous registers, new data is stored automatically on every rising edge of the clock.
- In most digital systems, the data is stored for several clock cycles before it is rewritten. For this reason it is useful to be able to control **WHEN** the data will be entered into a register.
 - Use a control signal called **Load** or **Enable**. This allows loading into a register known as a **parallel-load register**.

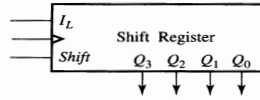
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```
Library ieee;
USE ieee.std_logic_1164.all;
ENTITY load_enable IS
    GENERIC ( N : INTEGER := 4);
    PORT ( D : IN STD_LOGIC_VECTOR (N-1 DOWNT0 0);
          Clock, Resetn, load : IN STD_LOGIC;
          Q : BUFFER STD_LOGIC_VECTOR (N-1 DOWNT0 0));
END load_enable;
ARCHITECTURE rtl OF load_enable IS
    SIGNAL state : std_logic_vector(N-1 DOWNT0 0);
BEGIN
    PROCESS (Resetn, Clock) IS
    BEGIN
        IF Resetn = '0' THEN
            state <= (OTHERS => '0');
        ELSIF (Clock'EVENT AND Clock = '1') THEN
            IF load = '1' THEN
                state <= D;
            ELSE
                state <= state;
            END IF;
        END IF;
    END PROCESS;

    Q <= state;
END rtl;
```

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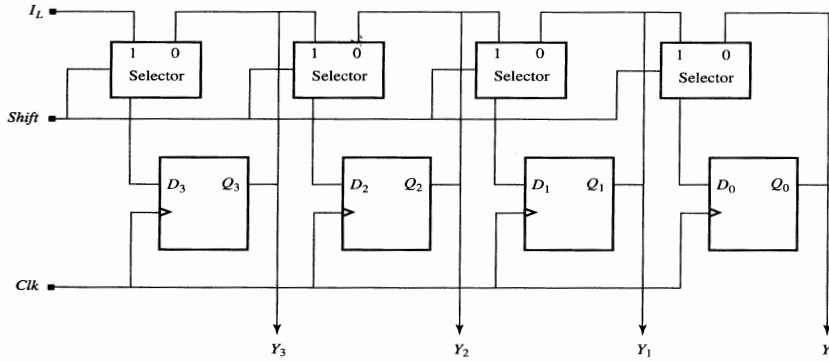
Serial-in/Parallel-out Shift Register



(a) Graphic symbol

PRESENT STATE	NEXT STATE			
Shift	Q_3	Q_2	Q_1	Q_0
0	No change			
1	I_L	Q_3	Q_2	Q_1

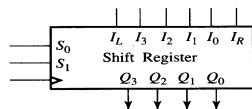
(b) Operation table



(c) Register schematic

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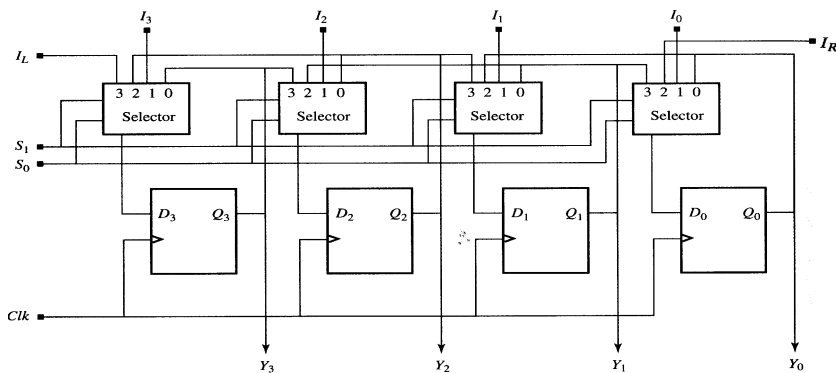
Parallel Load Shift Register



(a) Graphic symbol

PRESENT STATE		OPERATION	NEXT STATE			
S_1	S_0		Q_3	Q_2	Q_1	Q_0
0	0	No change	Q_3	Q_2	Q_1	Q_0
0	1	Load input	I_3	I_2	I_1	I_0
1	0	Shift left	Q_2	Q_1	Q_0	I_R
1	1	Shift right	I_L	Q_3	Q_2	Q_1

(b) Operation table



(c) Register schematic

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VHDL for Up-Counter

```

Library ieee;
USE ieee.std_logic_1164.all;
USE ieee.std_logic_unsigned.all;
ENTITY upcount IS
    GENERIC ( N : INTEGER := 4 );
    PORT ( Clock, Resetn, Enable : IN STD_LOGIC;
          Q: BUFFER STD_LOGIC_VECTOR (N-1 DOWNTO 0));
END upcount;

ARCHITECTURE cnt OF upcount IS
    SIGNAL count : STD_LOGIC_VECTOR (N-1 DOWNTO 0);

BEGIN
    PROCESS (Resetn, Clock)
    BEGIN
        IF Resetn = '0' THEN
            count <= ( OTHERS => '0' );
            -- Use of others in aggregate makes your code generic.
            -- Thus you won't have to replace all "00000000" with
            -- "00" when you change your mind and vectors should
            -- be only 2 bits wide.
        ELSIF (Clock'EVENT AND Clock = '1') THEN
            IF Enable = '1' THEN
                count <= count +1;
            ELSE
                count <= count;
            END IF;
        END IF;
    END PROCESS;

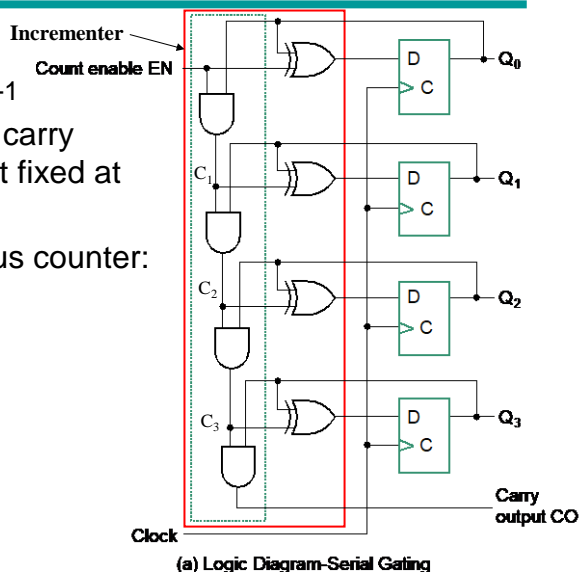
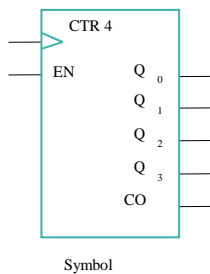
    Q <= count;
END cnt;

```

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Synchronous Counters

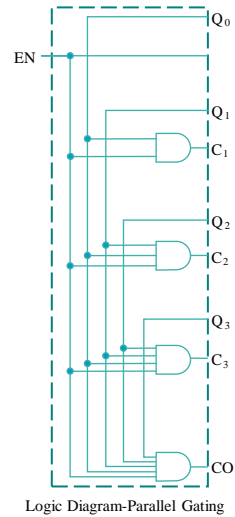
- Internal logic
 - Incrementer: $Q+0$ or $Q+1$
- Contraction of a ripple carry adder with one operand fixed at 000X
- Symbol for synchronous counter:



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Synchronous Counters (Contd.)

- Contraction of carry-lookahead adder
 - Reduce path delays
 - Called parallel gating
 - Lookahead can be used on COs and ENs to prevent long paths in large counters

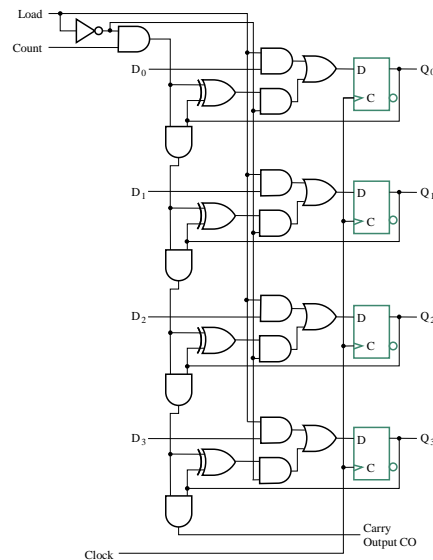
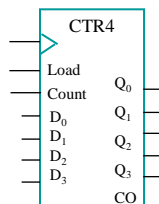


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Counter with Parallel Load

Load	Count	Action
0	0	Hold Stored Value
0	1	Count Up Stored Value
1	X	Load D

- Add path for input data D
 - enabled for Load = 1
- Add logic to:
 - When Load = 1 disable count logic (feedback from outputs)
 - When Load = 0 and Count = 1 enable count logic



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BCD Counter

```
architecture Behavioral of bcd_counter is
    signal regcnt : std_logic_vector(3 downto 0);
begin
    count: process (reset, clk) is
        begin
            if ( reset='1' ) then
                regcnt <= "0000";
            elsif ( clk'event and clk='1') then
                regcnt <= regcnt+1;
                if (regcnt = "1001") then
                    regcnt <= "0000";
                end if;
            end if;
        end process;
    end Behavioral;
```

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Overview

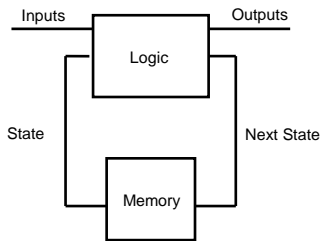
- Combinational circuits
 - Multiplexer, decoders, encoders, adders, comparators
- Sequential circuits
 - Regular sequential circuits
 - **Finite State Machines (FSMs)**

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FSM types

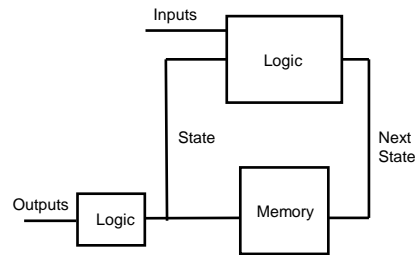
- **MEALY machine:**

- Outputs are dependent on current state and inputs



- **MOORE machine:**

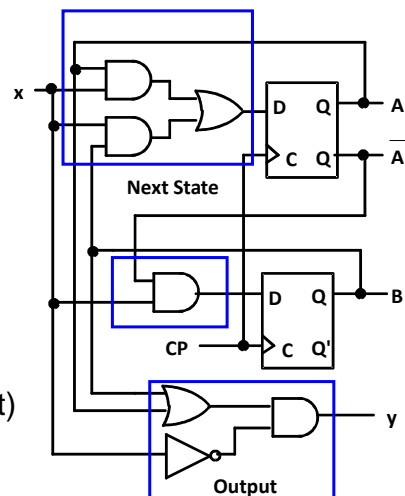
- Outputs are dependent on current state only



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Example 1

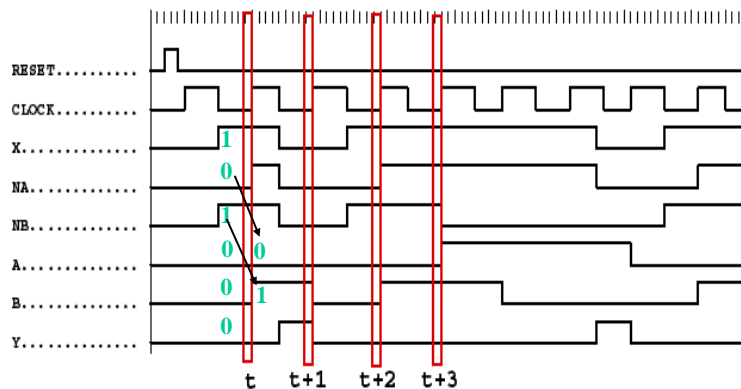
- Input: $x(t)$
- Output: $y(t)$
- State: $(A(t), B(t))$
- Output Function?
 - $y(t) = \overline{x(t)}(B(t) + A(t))$
- Next State Function?
 - $A(t+1) = A(t)x(t) + B(t)x(t)$
 - $B(t+1) = \overline{A(t)}x(t)$



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Example 1 (Contd.)

- Where in time are inputs, outputs and states defined?



$$y(t) = \overline{x(t)}(B(t) + A(t))$$

$$A(t+1) = A(t)x(t) + B(t)x(t)$$

$$B(t+1) = \overline{A(t)}x(t)$$

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Typical Design Procedure for Sequential Circuits

- Formulation:** Construct a state table or state diagram
- State Assignment:** Assign binary codes to the states
- Flip-Flop Input Equation Determination:** Select flip-flop types, derive flip-flop input equations from next state entries in the table
- Output Equation Determination:** Derive output equations from output entries in the table
- Optimization -** Optimize the equations
- Technology Mapping -** Find circuit from equations and map to flip-flops and gate technology
- Verification -** Verify correctness of final design

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Example 2: Sequence Recognizer

- A sequence recognizer: produces an output '1' whenever a prescribed pattern of inputs occur in sequence
- Steps:
 - Begin in an initial state (typically "reset" state), when NONE of the initial portion of the sequence has occurred
 - Add states
 - That recognize each successive symbol occurring
 - The final state represents the input sequence occurrence
 - Add state transition arcs which specify what happens when a symbol **not** in the proper sequence has occurred
 - Add other arcs on non-sequence inputs which transition to states
 - The last step is required because the circuit must recognize the input sequence regardless of where it occurs within the overall sequence applied since "reset"

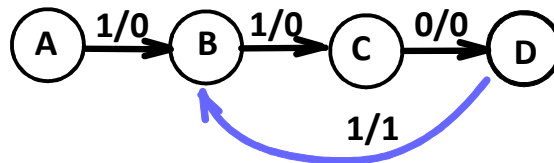
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Example 2: Recognize 1101 as Mealy machine

- Define states for the sequence to be recognized
- Starting in the initial state ("A"):



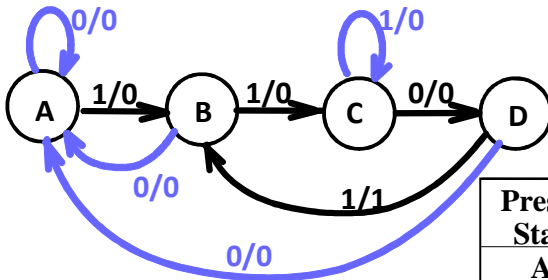
- Finally, output 1 on the arc from D means the sequence has been recognized,
 - To what state should the arc from state D go? Remember: 1101101 ?
 - The final 1 in the recognized sequence 1101 is a sub-sequence of 1101. It follows a 0 which is not a sub-sequence of 1101. Thus it should represent the same state reached from the initial state after a first 1 is observed.



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Example 2: Recognize 1101 (Contd.)

- The other arcs are added to each state for inputs not yet listed. Which arcs are missing?
 - State transition arcs must represent the fact that an input subsequence has occurred.
 - Note that the 1 arc from state C back to C implies that State C means *two or more 1's have occurred*.

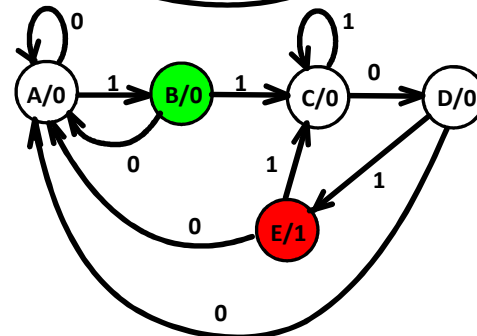
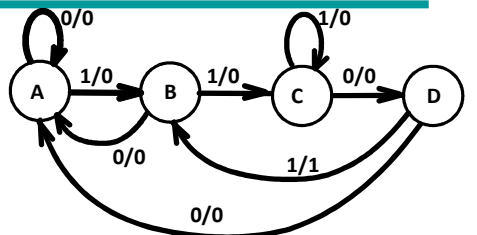


Present State	Next State		Output	
	x=0	x=1	x=0	x=1
A	A	B	0	0
B	A	C	0	0
C	D	C	0	0
D	A	B	0	1

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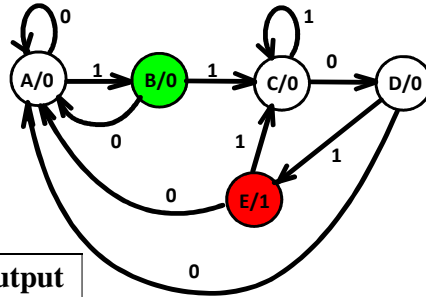
Example 2: Recognize 1101 as Moore machine

- For Moore Model, outputs are associated with states.** Arcs now show only state transitions
- Add a new state E to produce the output 1
 - State E produces the same behavior in the future as state B, but it gives a different output at the present time. Thus these states do represent a *different abstraction* of the input history.
- The Moore model for a sequence recognizer usually has *more states* than the Mealy model.



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Example 2: Moore Model (Contd.)



Present State	Next State		Output y
	x=0	x=1	
A	A	B	0
B	A	C	0
C	D	C	0
D	A	E	0
E	A	C	1

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VHDL code using 3 processes: sequential recognizer

```

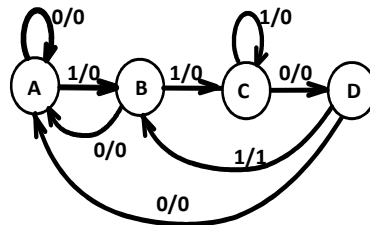
library ieee;
use ieee.std_logic_1164.all;

entity seq_rec_MEALY is
  port (CLK, RESET, X: in std_logic;
        Z: out std_logic);
end seq_rec;

architecture process_3 of seq_rec_MEALY is
  type state_type is (A, B, C, D);
  signal state, next_state: state_type;

begin

```



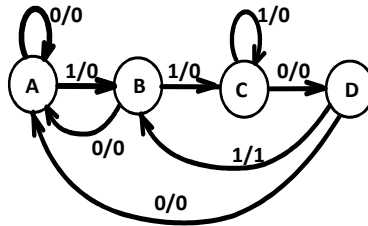
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```

-- process 1: implements positive edge-triggered
-- flipflop with asynchronous reset
state_register: process (CLK, RESET)
begin
  if (RESET = '1') then
    state <= A;
  elsif (CLK'event and CLK = '1') then
    state <= next_state;
  end if;
end process;

-- process 2: implement output as function
-- of input X and state
output_function: process (X, state)
begin
  case state is
    when A => Z <= '0';
    when B => Z <= '0';
    when C => Z <= '0';
    when D => if X = '1' then Z <= '1';
               else Z <= '0';
            end if;
  end case;
end process;

```

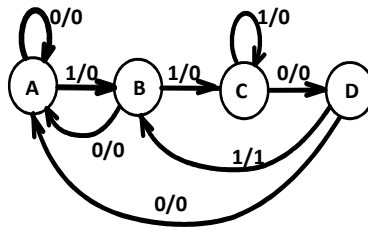


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```

-- process 3: next state-function implemented
-- as a function of input X and state
next_state_function: process (X, state)
begin
  case state is
    when A =>
      if X = '1' then next_state <= B;
      else next_state <= A;
      end if;
    when B =>
      if X = '1' then next_state <= C;
      else next_state <= A;
      end if;
    when C =>
      if X = '1' then next_state <= C;
      else next_state <= D;
      end if;
    when D =>
      if X = '1' then next_state <= B;
      else next_state <= A;
      end if;
  end case;
end process;
end architecture;

```



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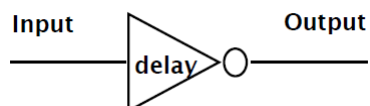
Summary

- Combinational circuits and regular sequential circuits are somewhat easier to describe in VHDL.
- Manual design of Finite State Machines becomes difficult for complex circuits. Automated tools come in handy in this case.

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Appendix A: VHDL Delay Models

- Delay is created by scheduling a signal assignment for a future time
- Delay in a VHDL cycle can be of several types:
 - Inertial
 - Transport
 - Delta

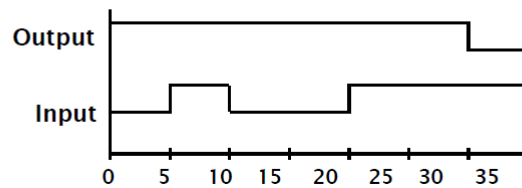
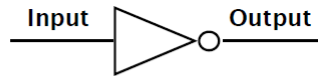


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Inertial Delay

- Default delay type
- Allows for user specified delay
- Absorbs pulses of shorter duration than the specified delay

```
-- Inertial is the default  
Output <= NOT Input AFTER 10 ns;
```

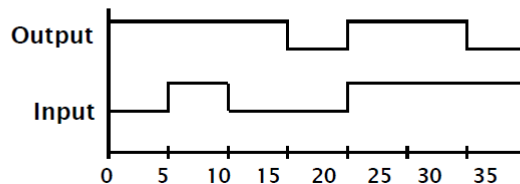
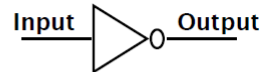


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Transport Delay

- Must be explicitly specified by user
- Allows for user specified delay
- Passes all input transitions with delay

```
-- TRANSPORT must be specified  
Output <= TRANSPORT NOT Input AFTER 10 ns;
```



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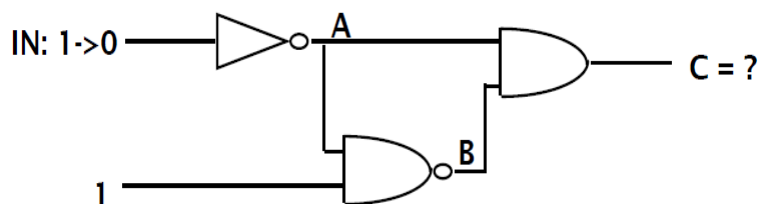
Delta Delay

- Delta delay needed to provide support for concurrent operations with zero delay
 - The order of execution for components with zero delay is not clear

- Scheduling of zero delay devices requires the delta delay
 - A delta delay is necessary if no other delay is specified
 - A delta delay does not advance simulator time or real/wall clock time
 - One delta delay is an infinitesimal amount of time
 - The delta is a scheduling device to ensure repeatability (or determinism)

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Example – Delta Delay



Using delta delay scheduling

Time	Delta	Event
0 ns	1	IN: 1->0
		eval inverter
	2	A: 0->1
		eval NAND, AND
	3	B: 1->0
		C: 0->1
		eval AND
	4	C: 1->0
1 ns		

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